Review of Matrix Theory with Applications in Education and Decision Sciences*

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Abstract

Matrix theory plays a very important role in teaching Mathematics and solving mathematical problems. Studying the theory of matrix can help academics, practitioners, and students solve many problems in Engineering, Econometrics, Finance, Economics, Optimization, Decision Sciences, and many other areas. To review the matrix theory with applications, in this paper we first review the theory of matrix. We then discuss how to build up some mathematical, financial, economic, and statistical models by using matrix theory and discuss the applications by using the theory of matrix in Decision Sciences and other related areas like Mathematics, Economics, Finance, Statistics, and Education with real-life examples.

Keywords: Matrix, Decision Sciences, Education and Reality.

JEL: A11, G02, G30, O35

1. Introduction

Matrix theory plays a very important role in teaching Mathematics and solving mathematical problems. Studying the theory of matrix can help academics, practitioners, and students solve many problems in Engineering, Econometrics, Finance, Economics, Optimization, Decision Sciences, and many other areas. Thus, many scholars have been studying the theory of matrix theory with its application. For instance, Gantmacher and Brenner (2005) presented applications of the theory of Matrices, Campbell and Meyer (2009) introduced the general inverses of linear transformations, Cullen (2012) studied the theory of matrix and linear transformations, and Farenick (2012) worked on the algebras of linear transformations by using matrices.

One of the main applications of matrix theory is to find the roots of equations and systems of equations. In this regard, Buttle (1967) presented the solution of coupled equations by using the R-matrix technique, El-Sayed and Ran (2002) provided an iteration method to solve different classes of nonlinear matrix equations, Saadatmandi and Dehghan (2010) introduced to a new operational matrix to solve fractional-order differential equations, and Doha, Bhrawy and Ezz-Eldien (2012) introduced a new Jacobi operational matrix with applications of solving fractional differential equations.

Matrix theory is useful in many scientific areas. For example, it has been applied in many different areas of physics like classical mechanics, optics, and quantum mechanics. Matrices have also been utilized in studying many physical phenomena, for example, the motion of rigid bodies. Moreover, in computer graphics, matrices have been used in developing 3D models and projecting them onto a 2-dimensional screen. In probability theory and statistics, stochastic matrices are used to depict sets of probabilities. Matrix calculation has been used in classical analytical concepts including derivatives and exponential functions for higher dimensions.

In addition, matrices can also be employed in economics to depict systems of economic relationships. For instance, Radhakrishna and Bhaskara (1998) presented the matrix algebra and its applications to statistics and econometrics, Calsbeek and Goodnight (2009) provided an empirical comparison of using G matrix test statistics to find biologically relevant change, Schott (2016) introduced the matrix analysis for statistics, and Magnus and Neudecker (2019) developed the matrix differential calculus with applications in statistics and econometrics.

To review the matrix theory with applications, in this paper we first review the theory of matrix. We then discuss how to build up some mathematical, financial, economic, and statistical models by using matrix theory and discuss the applications by using the theory of matrix in Decision Sciences and other related areas like Mathematics, Economics, Finance, Statistics, and Education with real-life examples.

This paper is organized as follows. We review the theory of matrix in Section 2. We present applications of matrix theory in Decision Sciences, Education and Reality in Sections 3, 4, and 5 and make some concluding remarks in the last section.

2. Review of Matrix Theory

In this section, we present the definitions of matrix and some common types of matrices

2.1. Definition of matrix

In Mathematics, a matrix is a rectangular array that contains numbers, symbols, or expressions, arranged in rows and columns, where each matrix follows predefined rules. Each cell in the matrix is called an element or an item. For instance, a matrix M with 2 rows and 4 columns and a matrix N with 3 rows and 1 column can be expressed as follows:

$$M = \begin{pmatrix} 1 & 0 & 7 & -1 \\ -2 & 3 & 9 & 5 \end{pmatrix} \text{ and } N = \begin{pmatrix} 2 & -4 \\ 5 & 6 \\ -3 & 1 \end{pmatrix}$$

When the numbers of columns and rows in a matrix are equal, the matrix is called a square matrix. We provided some common types of matrices in the next sub-section.

2.2. Common types of matrices

2.2.1. Zero matrix

A zero matrix is a matrix in which all its elements are zero numbers; that is, $m_{ij} = 0, \forall i, j$, and it is denoted by O_n or O.

Example 2.1

$$O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, O_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, O_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

2.2.2. Diagonal matrix

A diagonal matrix is a square matrix in which all elements outside the diagonal are zero; that is, $m_{ij} = 0, \forall i \neq j$. The diagonal matrix is symbolized by $M = diag(m_{11}, m_{22}, ..., m_{nn})$

Example 2.3

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \mathrm{M}_2 = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}, \mathrm{M}_3 = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}.$$

2.2.3. Identity matrix

An identity matrix is a diagonal matrix in which all the elements on the diagonal are equal to 1; that is, $m_{ii}= 1$. It is denoted by I or E. To specify the dimension of the matrix, it can be written as I_n or E_n.

Example 2.3

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathrm{I}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mathrm{I}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

2.2.4. Inverse matrix

Let M be a square matrix of dimension n, a square matrix N with dimension n is called an inverse matrix of matrix M if MN = NM = I, where I is an identity matrix. If M has the inverse matrix, then M is called an invertible matrix.

Example 2.4

If $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $N = \begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$, then MN = NM = I.

Thus, it can be said that M is an inverse matrix of N, or N is an inverse matrix of M.

2.2.5. Transpose matrix

The matrix $M^T = \left(m_{ji}\right)_{kh}$ is called a transpose matrix of $M = \left(m_{ji}\right)_{hk}$ if the rows of matrix M^T are the corresponding columns of matrix M and the columns of matrix M^T are the corresponding rows of matrix M.

Example 2.5

If $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then the transpose matrix of M is $M^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.

If $N = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, then the transpose matrix of N is $N^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$.

If $P = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, then the transpose matrix of P is $P^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$.

2.2.6. Conjugate transpose matrix

If M is an m × n matrix with entries from the field F, then the conjugate transpose of M is attained by first using the complex conjugate of each entry in M and then transposing M. A conjugate transpose matrix of $M=\left(m_{ij}\right)$ is $\overline{M}^{T}=\left(\overline{m}_{ji}\right)$. For the sake of simplicity, it is usually denoted by $\overline{M}^{T}=M^{*}$.

Example 2.6

If $M=\begin{pmatrix} 1 & -2i \\ 1-3i & 4 \\ 3 & 3+2i \end{pmatrix}$, then $\overline{M}=\begin{pmatrix} 1 & 2i \\ 1+3i & 4 \\ 3 & 3-2i \end{pmatrix}$ and $M^{*}=\overline{M}^{T}=\begin{pmatrix} 1 & 1+3i & 3 \\ 2i & 4 & 3-2i \end{pmatrix}$

Thus, M is a conjugate transpose matrix.

2.2.7. Orthogonal matrix

A square matrix M is called an orthogonal matrix if $M^{T}M=MM^{T}=I$, where I is an identity matrix and M^{T} is the transpose matrix of M. This leads to the following equivalent equality: M is an orthogonal matrix if its inverse is equal to its transpose; that is, $M^{-1}=M^{T}$.

Example 2.7

If $M=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, then $M^{-1}=M^{T}=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$.Thus, M is an orthogonal matrix.

2.2.8. Unitary matrix

M is called a unitary matrix if $M^{*}M=I$, where I is an identity matrix and M^{*} is a conjugate transpose matrix of M. If M is a unitary matrix, then M is invertible. This leads to the following equivalent equality: If M is an orthogonal matrix, then its inverse is equal to its conjugate transpose; this is, $M^{-1}=M^{*}$. Furthermore, $\det\left(M\right)=\pm 1$ where $\det\left(M\right)$ is the determinant of M because it is well known that $\det\left(I\right)=\det\left(M^{*}M\right)=\det\left(M^{*}\right)\det\left(M\right)=\left[\det\left(M\right)\right]^{2}=1$.

Example 2.8

If $M = \begin{pmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{pmatrix}$, then $\overline{M} = \begin{pmatrix} \frac{1-i}{2} & \frac{-1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix}$ and $M^* = \overline{M}^T = \begin{pmatrix} \frac{1-i}{2} & \frac{1-i}{2} \\ \frac{-1-i}{2} & \frac{1+i}{2} \end{pmatrix}$

It can be seen that $M^*M = I$. Thus, M is a unitary matrix.

2.2.9. Symmetric matrix

If all the elements of a matrix $M_{n\times n}$ are real and satisfy the condition that $M = M^T$, then it is called a symmetric matrix.
We note that if $M_{n\times n}$ is a symmetric matrix, then $M_{\text{ij}} = M_{ji}, \forall i, j = 1, 2, ..., \text{n}$.

Example 2.9
If $M = \begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix}$, then $M^T = M = \begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix}$. Hence, M is a symmetric matrix.
If $N = \begin{pmatrix} 2 & 5 & 7 \\ 5 & -3 & 4 \\ 7 & 4 & -1 \end{pmatrix}$, then $N^T = N = \begin{pmatrix} 2 & 5 & 7 \\ 5 & -3 & 4 \\ 7 & 4 & -1 \end{pmatrix}$. Hence, N is a symmetric matrix.

2.2.10. Hermitian matrix

A matrix M is called a Hermitian matrix if $M = \overline{M}^T$.

Example 2.10
If $M = \begin{pmatrix} 1 & 3+2i \\ 3-2i & 4 \end{pmatrix}$, then $\overline{M} = \begin{pmatrix} 1 & 3-2i \\ 3+2i & 4 \end{pmatrix}$ and $\overline{M}^T = \begin{pmatrix} 1 & 3+2i \\ 3-2i & 4 \end{pmatrix}$

It can be seen that $M = \overline{M}^T$. Thus, M is also Hermitian matrix.

2.2.11. Positive semi-definite matrix

The quantity

$$u^T Mu = \sum_{i=1}^{n}\sum_{j=1}^{n} m_{\text{ij}} u_i u_j$$

is called the quadratic form. A Hermitian matrix $M \in C^n$ is said to be positive semi-definite (PSD) if $u^T Mu \geq 0$, for any $u \in C^n$ and $u \neq 0$.

Example 2.11

If $M = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$, then $u^T Mu = \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 4u_1^2 + 2u_2^2 \geq 0.$

Thus, M is a positive semi-definite matrix.

2.2.12. Positive definite matrix

A Hermitian matrix $M \in C^n$ is said to be positive semidefinite (PSD) if $u^T Mu > 0$ for any $u \in C^n$ and $u \neq 0.$ The illustration for the quadratic form for a positive definite matrix is provided in Figure 1 and the following example:

Example 2.12

If $M = \begin{pmatrix} 7 & 0 \\ 0 & 9 \end{pmatrix}$, then $u^T Mu = \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & 9 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 7u_1^2 + 9u_2^2 \geq 0,$ since $u \neq 0.$

Thus, M is a positive definite matrix.

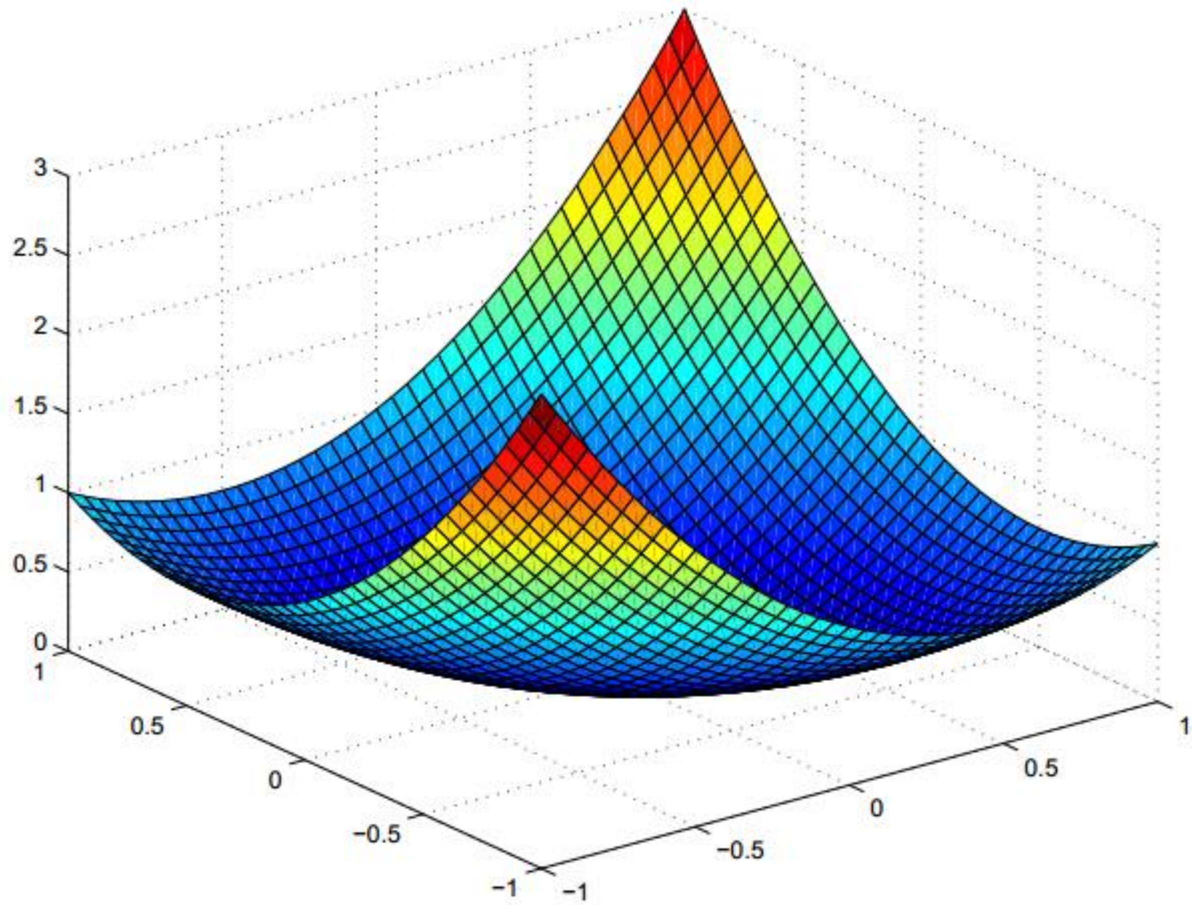

Figure 1: Quadratic form for a positive definite matrix.

2.2.13. Indefinite matrix

A Hermitian matrix that is not PD or PSD is called an indefinite matrix. The illustration for the quadratic form for an indefinite matrix is provided in Figure 2 and the following example:

Example 2.13

If $M = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, then $u^T M u = \begin{pmatrix} u_1 & u_2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 2u_1^2 - 3u_2^2$.

Thus, M is an undefinite matrix.

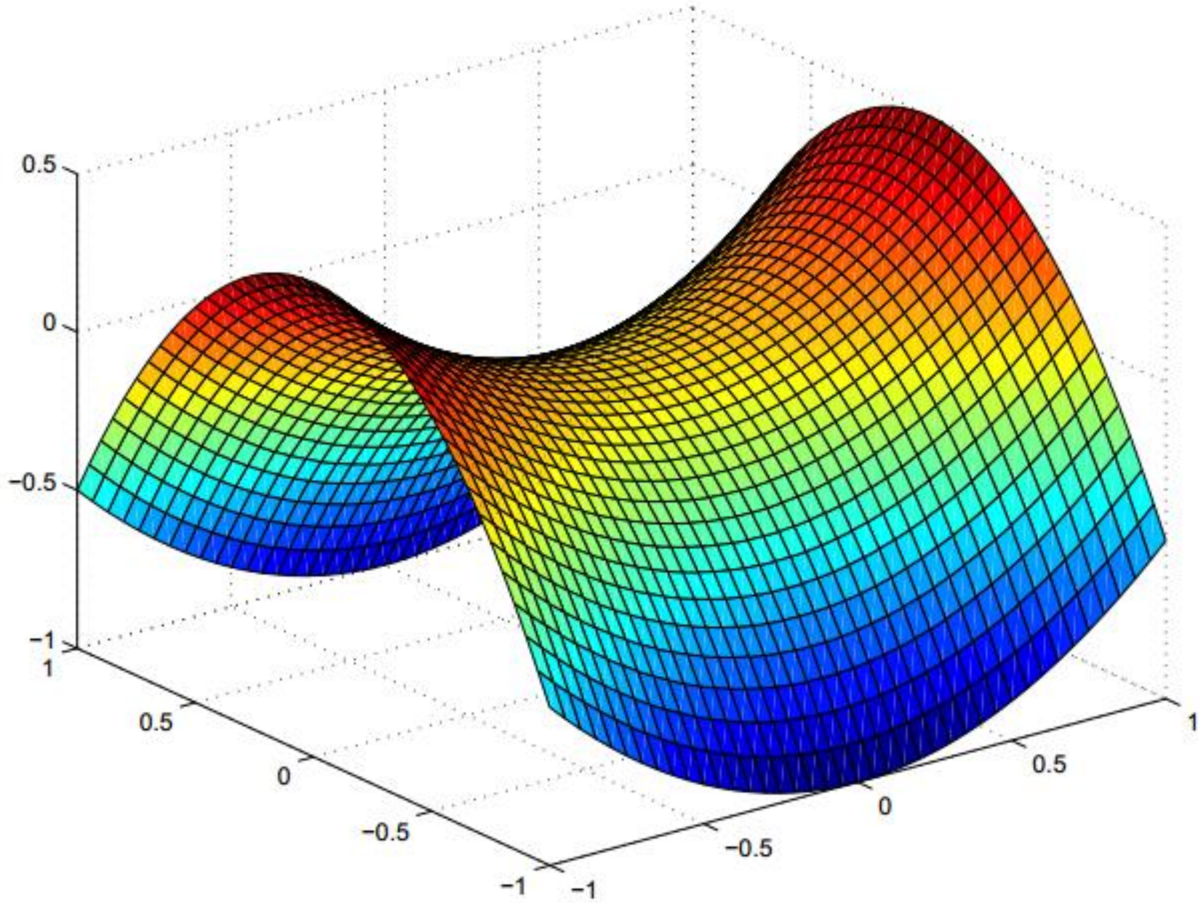

Figure 2: Quadratic form for an indefinite matrix.

3. Applications of Matrix Theory in Decision Sciences

There are many applications of matrix theory in Decision Sciences, especially in Applied Mathematics, Finance, and Economics because matrix theory is not only the foundation of Mathematics and Statistics but also the foundation of all sciences. In this section, we will present the applications of matrix theory in four main areas, including Applied Mathematics, Statistics, Finance, and Economics. We first present the applications of matrix theory to Applied Mathematics.

3.1. Building up mathematical models

The main area of applying Matrix Theory in Applied Mathematics is to solve systems of equations. There are two approaches in solving systems of equations: the Newton method and Cramer's formula. For instance, Pho, et al. (2018) compared the Newton algorithm and maxLik function and introduced some applications of this approach in Statistics and regression models with missing data. Truong, et al. (2019) introduced some applications of the Newton method in Decision Sciences and Education.

In addition, Pho, et al. (2019a) introduced the moment generating function, expectation and variance of ubiquitous distributions with applications in decision sciences. Pho, et al. (2019b) applied regression models in some applications by using the Newton method and Pho. et al. (2019c) reviewed the four optimal solution methods in decision sciences: bisection, gradient, secant, and Newton methods. On the other hand, Truong, et al. (2019) used optim, nleqslv, and maxLik to determine parameters in some of regression models, these are useful functions in R programmed by the Newton method, etc.

For the Cramer formula, it is named after the famous mathematician Gabriel Cramer (1704-1752) who proposed the rule for an arbitrary number of unknowns in 1750. This is a very popular formula to solve the system of equations that there have been many scientists studying and improving the formula recently. For instance, Burgstahler (1983) presented a more general Cramer Rule, Chen (1993) provided a Cramer rule for root of general restricted linear equations, and Wei (2002) offered a characterization for the W-weighted Drazin inverse and a Cramer rule for the W-weighted Drazin inverse solution.

On the other hand, Ji (2005) introduced explicit expressions of the generalized inverses and condensed Cramer rules and Song et al. (2011) presented the Cramer rule for the unique root of restricted matrix equations over the quaternion skew field. Egozcue, et al. (2009, 2011a,b, 2012a, 2013) and Ly, et al. (2019a,b) applied the theory of matrix to develop some new theories for covariance and copulas. Readers may refer to Wang and Xu (2005), Jiang (2006), and Akhtyamov, et al. (2017) for more discussions.

3.2. Building up financial models

There are many applications of matrix theories in Finance. For instance, Norberg (1999) presented on the Vandermonde matrix and its role in mathematical finance. Laloux et al. (2000) and Bai et al. (2011a) provided the random matrix theory and financial correlations. Bai et al. (2009a,b, 2016) and Li et al. (2020) applied the theory of random matrix to develop efficient estimates of portfolios. Higham (2002) developed the computing the nearest correlation matrix to a problem from finance. Ledoit and Wolf (2003) offered the improved estimation of the covariance matrix of stock returns with an application to portfolio selection.

In addition, Dourson (2004) provided the 40 inventive principles of TRIZ applied to finance. Zhao et al. (2011) introduced to synchronization of a chaotic finance system. Zhaoben et al. (2014) introduced to spectral theory of large dimensional random matrices and its applications to wireless communications and finance statistics: random matrix theory and its applications. Wong and Chan (2004), Thompson and Wong (1996, 1991), and others applied the theory of matrix theory to extend the theory of cost of capital that allows dividends to a time series model.

Furthermore, Guo, et al. (2017), Fung, et al. (2011), Lam, et al. (2010, 2012), and others applied the theory of matrix theory and applied the theory of cost of capital developed by Wong and Chan (2004), Thompson and Wong (1996, 1991), and others to develop a pseudo Bayesian model that can explain investors' behaviors and explain market anomalies like under- and overreaction and market volatility. Lozza, et al. (2018), Egozcue, et al. (2011), Egozcue and Wong (2010), and many others applied the theory of matrix to develop the theory of diversification and Guo, et al. (2019) applied the theory of matrix to develop a new portfolio theory with background risk.

3.3. Building up economic models

Lewis and Thorbecke (1992) presented district-level economic linkages in Kenya: evidence based on a small regional social accounting matrix. Sonis and Hewings (1999) provided economic landscapes: multiplier product matrix analysis for multiregional input-output systems. Ormerod and Mounfield (2000) introduced to random matrix theory and the failure of macro-economic forecasts. Merkesdal et al. (2001) offered the development of a matrix of cost domains in an economic evaluation of rheumatoid arthritis.

In addition, Lee et al. (2006) developed the economic value portfolio matrix: A target market selection tool for destination marketing organizations. Morilla et al. (2007) presented Economic and environmental efficiency using a social accounting matrix. Massetti (2008) proposed the social entrepreneurship matrix as a "tipping point for economic change. Allan et al. (2011) offered the importance of revenue sharing for the local economic impacts of a renewable energy project: a social accounting matrix approach, etc.

The theories of stochastic dominance (SD) for risk averters and risk seekers in a univariate dimension developed by Wong (2006, 2007), Wong and Li (1999), Levy (2015), Li and Wong (1999), and others have been well established. Guo and Wong (2016) applied the theory of matrix theory to extend the theory to multiple dimensions.

Similarly, the SD theories for investors with (reverse) S-shaped utility functions developed by Wong and Chan (2008), Levy and Levy (2002, 2004), and others and the theory of almost SD developed by Guo, et al. (2013, 2014, 2016) and others in univariate dimension are well established. One could follow the approach of applying the theory of matrix theory by Guo and Wong (2016) to extend the SD theories for investors with (reverse) S-shaped utility functions and the theory of almost SD to multiple dimensions.

Wong and Ma (2008) applied the theory of matrix to extend the work by Wong (2006, 2007) and others on the theory of indifference curve to include location-scale family in a multivariate setting for both risk averters and risk seekers. One could easily use the same approach introduced by Wong and Ma (2008) to extend the work by Broll, et al. (2010) and others to obtain the theory of indifference curve for investors with (reverse) S-shaped utility functions in a multivariate setting. In addition, Guo, et al. (2018) applied the theory of matrix to develop a new statistic for the stochastic frontier model.

3.4. Building up statistical models

Many problems in statistics, especially for statistics in multi-dimensional cases, are related to matrix theory. The applications of matrix theory in Statistics are immensely diverse and varied. For example, Muttalib, et al. (1987) presented the theory of random matrix and universal statistics for disordered quantum conductors, Zanon and Pichard (1988) introduced the theory of random matrix and universal statistics for disordered quantum conductors with spin-dependent hopping, and Searle and Khuri (2017) presented the matrix algebra that is useful for statistics.

On the other hand, Andreev and Altshuler (1995) developed the spectral statistics beyond the theory of random matrix, Tulino and Verdu (2004) applied the theory of random matrix to wireless communications. Bai, et al. (2009a, b) used the theory of large sample matrix (Bai, et al. (2011a) to correct the overestimation problem for portfolio optimization first developed by Markowitz (1952), Leung, et al. (2012) introduced the formula with a closed form for the estimation theory developed by Bai, et al. (2009a, b), and Li, et al. (2020) further extended the theory developed by Bai, et al. (2009a, b) by introducing the spectralcorrected estimation that outperforms the estimation proposed by Bai, et al. (2009a, b). Li, et al. (2018) extend the theory to take care of investors preferences.

The theories of stochastic dominance (SD) for risk averters and risk seekers have been developed by Wong (2006, 2007), Wong and Li (1999), Levy (2015), Guo and Wong (2016), Li and Wong (1999), Chan, et al. (2020), Sriboonchitta, et al. (2009), and many others while Bai, et al. (2015), Ng, et al. (2017), and others applied the theory of matrix theory to develop SD tests for risk averters and risk seekers and Lean, et al. (2008), Ng, et al. (2017), and others have shown that SD tests developed by Bai, et al. (2015) and Ng, et al. (2017) are efficient and have decent size and good power.

The SD theories for investors with (reverse) S-shaped utility functions have been developed by Wong and Chan (2008), Levy and Levy (2002, 2004), and others while Bai, et al. (2011) applied the theory of matrix theory to develop SD tests for investors with (reverse) S-shaped utility functions. On the other hand, Guo, et al. (2013, 2014, 2016) and others have developed the theory of almost SD. One could easily apply the theory of matrix theory to develop tests for almost SD.

Leung and Wong (2008), Wong, et al. (2012), and Bai, and et al. (2011, 2012) applied the theory of matrix to extend the Sharpe ratio in which Leung and Wong (2008) developed a test for multiple Sharpe ratios, Wong, et al. (2012) develop a mixed Sharpe ratio, Bai, and et al. (2012) develop a mean-variance ratio test, and Bai, et al. (2012) extend the test developed by Bai, et al. (2011) to obtain a non-asymptotic UMPU test.

In addition, Niu, et al. (2018) applied the theory of matrix to develop the theory of confidence interval for the economic performance measure. One could easily apply the theories developed by Leung and Wong (2008), Wong, et al. (2012), Bai, and et al. (2011, 2012), and Niu, et al. (2018) to develop the tests and confidence intervals for the mean-variance rule (Wong and Ma (2008), moment rule (Chan, et al., 2019), Copulas (Tang, et al., 2014; Ly, et al., 2019a,b), VaR and conditional-VaR (Ma and Wong, 2010; Guo, et al. 2019), Kappa ratio (Niu, et al., 2017), Omega ratio (Guo, et al., 2017), Farinelli and Tibiletti ratio (Guo, et al., 2019), and many others.

After Guo, et al. (2017), Fung, et al. (2011), Lam, et al. (2010, 2012), and others applied the theory of matrix theory to develop a pseudo-Bayesian model that can explain market anomalies like under- and overreaction and market volatility, Fabozzi, et al. (2013), Lam, et al. (2008), and others applied the theory of matrix theory to develop some statistics that can be used to measure some market anomalies like under- and overreaction and market volatility.

Tiku and Wong (1998) is the first paper that applies the theory of matrix theory and time series analysis to obtain robust estimation for unit root, Tiku, et al. (2000, 1999) extend the theory to get a robust estimation for AR(q) models, while Tiku, et al. (1999a) and Wong and Bian (2005) further extend the theory to regression with AR(1) innovations in which the innovations are distributed as symmetric t-distribution and asymmetric gamma and generalized logistic distributions, respectively. Bian and Wong (1997) applied the theory of matrix theory to develop a (g-prior) Bayesian regression model by using Cauchy and inverted gamma prior.

They find that their proposed estimator is adaptive and uniformly better than the leastsquares (LS) estimators. Bian, et al. (2013) applied the g-prior Bayesian regression model developed by Bian and Wong (1997) in the estimation of the Capital Asset Pricing Model (CAPM) on the returns of several US portfolios. They find that the g-prior estimators are more efficient than and outperform the LS estimator uniformly. Readers may refer to Kottos (2005), Paul and Aue (2014), Lambert, et al. (2018), Tian et al. (2019), Tuan et al. (2019a b, c), Fu et al. (2020), Mahmoudi, et al. (2020a, b, c, d, e), Seyed et al. (2020), Wang et al. (2020) and Pho, et al. (2020a, b), Zhou et al. (2020) for more discussion.

Besides, Guo, et al. (2018) applied the theory of matrix to develop a new statistic for the stochastic frontier model, Bian, et al. (2011) applied the theory of matrix to develop a new trinomial test, Wong and Miller (1990) and Wong, et al. (2001) applied the theory of matrix to develop a new time series model, Lam, et al. (2006) applied the theory of matrix to develop a new variance ratio test, Tiku and Wong (1998) applied the theory of matrix to develop a new unit root test, Penm, et al. (2003) and Wong, et al. (2007) applied the theory of matrix to develop a new cointegration test, Bai, et al. (2018, 2010, 2011) and Chow et al. (2018) applied the theory of matrix to develop new causality tests, and Hui, et al. (2017) applied the theory of matrix to develop a new nonlinearity statistic.

3.5. Applications of mathematical, economic, financial, and statistical models

After applying the theory of matrix theory to develop the mathematical, economic, financial, and statistical models as discussed in the previous subsections, one may consider employing the model to analyze some real-life problems. For example, after applying the theory of matrix theory to develop the mathematical, economic, financial, and statistical models as discussed in the previous subsections, one may consider employing the model to analyze some real-life problems.

For example, after applying the theory of random matrix to developing models of portfolio optimization, Bouri, et al. (2018), Hoang, et al. (2018, 2019, 2015a,b), Abid, et al. (2014, 2013, 2009), Mroua, et al. (2017), and many others applied the theory of portfolio optimization to analyze many interesting financial issues.

There are many good empirical applications of the work after applying the theory of matrix, for example, in two-moment model (Broll, et al., 2011, 2006, 2015; Guo, et al., 2018; Alghalith, et al., 2016, 2017, 2017a), production model (Guo, et al., 2015, 2020, Egozcue, et al., 2015; Guo and Wong, 2020), currency model (Agarwal, et al., 2004; Chen, et al., 2011), risk measures (Bai, et al., 2013; Chow, et al. 2019), cointegration and causality (Batai, et al., 2017; Chen, et al., 2007, 2008; Cheng, et al., 2019; Chow, et al., 2019, Liew, et al., 2010; Lv, et al., 2019; Qiao, et al., 2009; Zheng, et al., 2009), stochastic dominance (Bouri, et al., 2018).

In addition, income inequality (Chan, et al., 2018; Valenzuela, et al., 2019), technical analysis (Chan, et al., 2014; Chong, et al., 2017; Kung and Wong, 2009a,b; Lam, et al., 2007; Wong, et al., 2001, 2003, 2005), trading strategies (Lu, et al., 2018, 2019; McAleer, et al., 2016), and calendar anomalies (Wong, et al., 2004a, 2006; Lean, et al., 2007; Qiao, et al., 2010).

There are many applications in different financial markets as well, for example, warrant markets (Chan, et al., 2012; Wong, et al., 2018), stock markets (Demirer, et al., 2019; Fong, et al., 2008; Qiao, et al., 2008a,b, 2011; Wong and Bian, 2000; Xu, et al., 2017, Wong, et al., 2004), future markets (Clark, et al., 2016; Lam, et al., 2016; Qiao, et al., 2012, 2013; Lean, et al., 2010, 2015), bond (Kung and Wong, 2006; Liew, et al., 2010; Chow, et al., 2019), currency (Owyong, etala 2015; Agarwal, et al., 2004), and housing markets (Qiao, and Wong, 2015; Tsang, et al., 2016; Lam, et al., 2016; Li, et al., 2014).

Matrix theory could also be used in marketing (Liao, et al., 2012, 2014; Liao and Wong, 2008; Moslehpour, et al., 2017a,b; 2018) and management (Pham, et al., 2020). There are many other good applications of applying the theory of matrix, see, for example, Chang, et al. (2017, 2016a,b,c, 2018a,b), Pho, et al. (2019a, b), Truong, et al. (2019), Woo, et al. 2020) for more information.

4. Applications of Matrix Theory in Education

The theory of practical Mathematics related to Matrix theory becomes an important problem in some Mathematics competitions in Vietnam, including National High School, College entrance exam, National Student Mathematics Olympiad, and others. Teaching Mathematical Modeling becomes effective for High School teachers as well as University lecturers/professors. Through the activities of teaching Mathematical Modeling, students get a better understanding of all problems related to Mathematics they encountered.

In Vietnam, Nam (2015a) presented some design modeling activities in teaching mathematics, Nam (2015b) provided the process modeling in teaching mathematics in high school, and Nam (2015c) introduced the capacity of mathematical modeling for High School students. In addition, Nam (2016) developed some modeling capabilities for High School teachers.

Many scientists have been studying problems related to teaching Mathematical Modeling in different countries. For instance, Burghes and Huntley (1982) presented the teaching mathematical modeling on reflections and advice while Verschaffel and De Corte (1997) provided approaches in teaching realistic mathematical modeling in the elementary school with teaching experiment on some fifth-grade students. Also, Abrams (2001) discussed the teaching mathematical modeling and provided some skills of representing mathematical modeling.

Besides, Lingefjard and Holmquist (2005) introduced a theory on assessing students' attitudes, skills and competencies in mathematical modeling, Lesh et al. (2010) presented problems on students mathematical modeling competencies, and Erbas et al. (2014) provided the basic concepts and approaches to the Mathematical Modeling in Mathematics Education. Readers may refer in English (2006), Mousoulides, Christou and Sriraman (2008), and Clarke and Skiba (2013) for more information.

In order to help teachers and students in high schools and universities to have a good review on teaching Mathematical Modeling, in this paper we review the following issues: Mathematical modeling capabilities of High School teachers, the capacity of Mathematical modeling of High School students, the effective way of teaching through Mathematical modeling, and the Mathematical modeling step for the practical problem about Matrix theory.

4.1. Mathematical modeling capabilities for High School teachers

Nam (2016) studied Modeling capabilities for High School teachers. He pointed out that the ability of Vietnamese High School teachers in teaching Mathematical modeling is still weak and the facilities and teaching equipment for teaching Mathematical modeling are not up to standard. Thus, it is important to help Vietnamese High School teachers to improve their ability and help them to get more and better resources in teaching Mathematical modeling. Many studies have been working in this area.

For example, Jaworski (2004) presented the design and study of classroom activity in mathematics teaching development include insiders and outsiders. Silk et al. (2010) designed technology activities that can be used in teaching mathematics. Shaffer (2013) studied design, collaboration, and computational model with computer-supported collaboration in mathematics. In addition, Nam (2015a) designed modeling activities in teaching mathematics and Watson, et al. (2015) introduced to task design in mathematics education.

4.2. The capacity of Mathematical modeling for High School students

Nam (2015c) studied the capacity of mathematical modeling for High School students in Vietnam. He pointed out that there is not enough capacity for mathematical modeling for High School students in

Vietnam and most of the capacity is only at a low level. He also pointed out that the limitation of the current textbook curriculum, in which the applicability of mathematics in solving daily-life problems has not been taken seriously. To circumvent the limitations of teaching mathematical modeling for High School students in Vietnam, Nam (2015c) suggested the way to teach Mathematical Modeling in high schools should focus on training students to achieve competence in mathematical modeling with emphasis on solving daily-life problems.

4.3. The effective way of teaching through Mathematical modeling

To provide an effective way of teaching through Mathematical modeling, Nam (2015b) proposed an approach to organizing model activities in teaching Mathematics. We summarize his approach in detail and fully by using Figures 3 and 4 to show a comprehensive overview of the effective way of teaching through Mathematical modeling. We note that teachers should use the practical problems that are useful to students and easy to learn. The easier the problem that can be used in practice, the easier and faster students could learn the theory.

It can be seen from Figure 3 that, using Mathematical modeling will help students solve problems by collecting, understanding and analyzing information about Mathematics first. Students can then apply Mathematics to model real-world situations. However, in teaching practice, the Mathematical modeling process above always follows an appropriate adjustment mechanism to simplify and make the problem easier for students to understand. This adjustment mechanism demonstrates the close relationship between Mathematics and practical problems as stated in Figure 4.

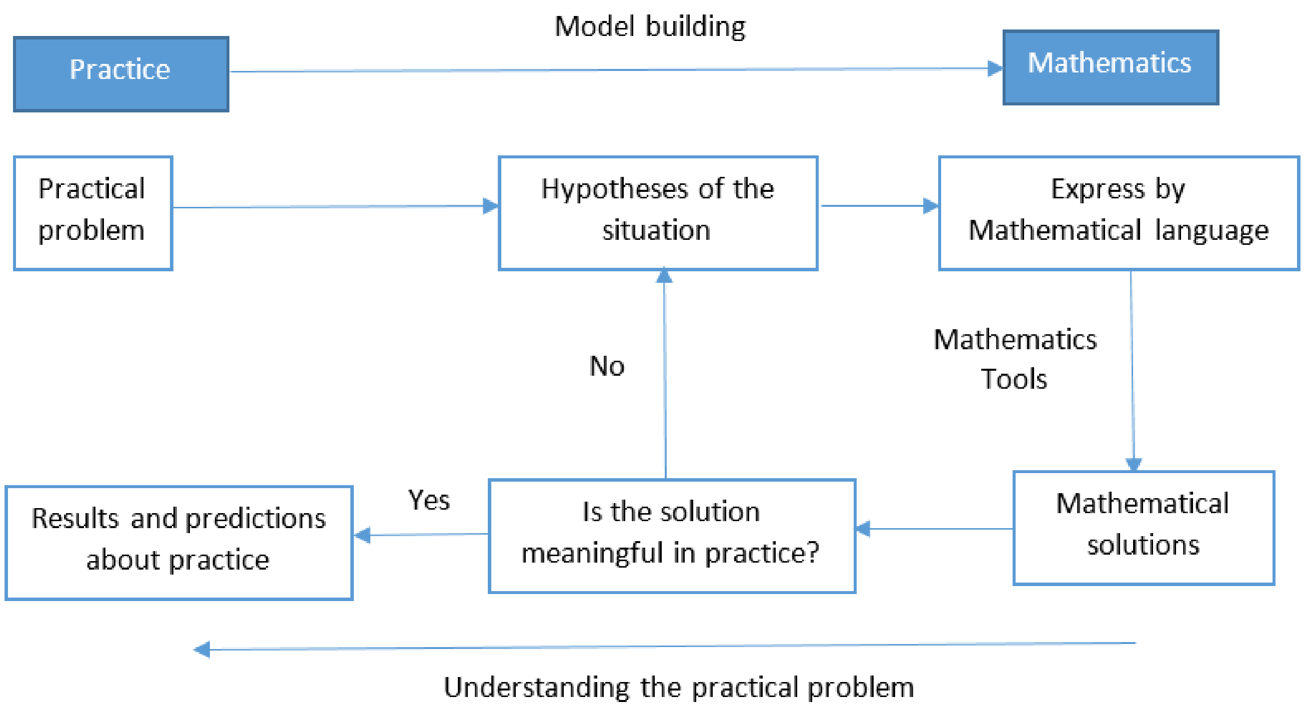

Figure 3: The process of Mathematical Modeling Teaching.

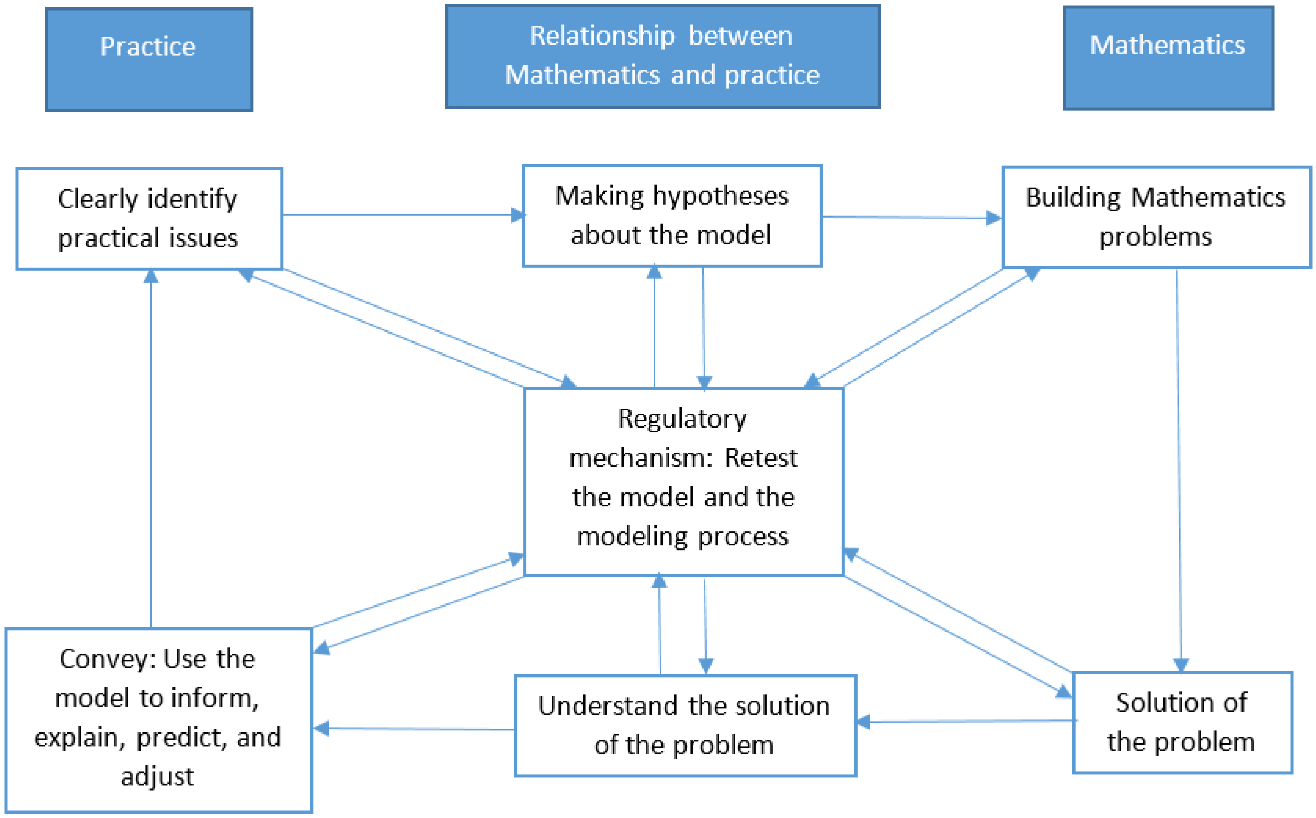

Figure 4: Mechanism to adjust the modeling process.

4.4. The Mathematical modeling step for the practical problem about Matrix theory

In this section, we suggest the following steps to solve the problem by setting the system of equations:

Step 1: Select the unknown variable and set the condition for the unknown variable because usually, the unknown variable is the quantity of the problem to be found. Then, represent unknown quantities according to unknown variables and known quantities. Finally, program equations (systems of equations) to denote the relationship between quantities.

Step 2: Solve the equation (system of equations). Usually, the problem will lead to a system of equations with many unknown variables. If the problem is written in a matrix form, then the system of equations can be written in the form: AX = B. Then, we can easily find the solution to the system of equations: X $=A^{-1}B$.

Step 3: Conclusion: It is important to check that in the solutions of the equation to get solutions that satisfy the conditions of the unknown variable and find out which solutions do not satisfy and draw a conclusion.

5. Applications of Matrix Theory in Reality

The National Student Mathematics Olympiad is a very important examination for university students in Vietnam because the contest not only improves the quality of teaching and learning Mathematics but also gives a good motive to students to learn mathematics. The competition is organized to attract students in universities, colleges, and institutes who are good at mathematics to participate.

In addition, the competition also creates opportunities for interactions among students, teachers, and professors from universities, colleges, and institutes. In order to provide readers to have a good overview of the applications of matrix theory in reality, in this paper we use the theory of Matrix theory to gather, solve, and analyze some practical problems. These problems are illustrated in the National Student Mathematics Olympiad in 2015, 2016, 2017 and 2019.

5.1. Practical Mathematics problems in the National Student Mathematics Olympiad

Problem 1: (The test of the National Student Mathematics Olympiad in 2016)

Let a and b be real numbers and

$$A=\begin{pmatrix} -a & b & 0 & 0 \\ 0 & -a & b & 0 \\ 0 & 0 & -a & b \\ b & 0 & 0 & -a \end{pmatrix}$$

(i) Find the determinant of A.

(ii) For which values of a and b, A is invertible and in that case, compute A^{-1}.
(iii) The urban green company implements the Project to replace old and out-of-species trees with new ones. The company carried out the program for four months. In each month the company will cut 10% of the total trees in the city by the first day of the month, at the same time, planting some more trees. Specifically, in the first month, 100 trees will be planted, 102 more trees in the second month, 104 trees in the third month, 106 trees in the last month. At the Closing Ceremony, it is said that the total number of existing trees in the city has increased by 80 compared to before Project implementation. How many trees are there in the city now?

Problem 2: (The test of the National Student Mathematics Olympiad in 2019)

A factory produces five types of products A, B, C, D, E. Each type of product must go through five stages of cutting, trimming, packing, decorating and labeling with a time for each stage as shown in the following table:

	Cutting (hours)	Trimming (hours)	Packing (hours)	Decorating (hours)	Labeling (hours)
Product A	1	1	1	1	1
Product B	4	3	3	2	1
Product C	8	12	6	3	1
Product D	12	15	10	4	1
Product E	20	24	10	5	1

Suppose that cutting, trimming, packing, decorating, and labeling have the maximum number of hours a week, respectively, 180, 220, 120, 60, and 20 hours. In the original design of the plant, there was a plan on the number of each type of product the factory had to produce in a week to use up the capacity of the parts. Calculate the quantity of each type of product produced in a week according to that plan.

5.2. Solving problems

In this study, our main purpose is to show how to use Matrix theory to solve practical problems, so we do not present detailed solutions.

Solution to Problem 1:
(i) The determinant of A will be equal to $a^4 - b^4$.
(ii) If $a \neq b$, then A is invertible. Using Cramer formula, we can easily find the following result:

$$A^{-1} = \left(b^4 - a^4\right)^{-1} \begin{pmatrix} a^3 & a^2b & ab^2 & b^3 \\ b^3 & a^3 & a^2b & 0 \\ ab^2 & b^3 & a^3 & a^2b \\ a^2b & ab^2 & b^3 & a^3 \end{pmatrix}$$

(iii) To solve the problem by setting up the system of equations, we proceed through three steps as follows:

Step 1: Let y_0 be the original number of trees is managed by the company. Let y_1, y_2, y_3 and y_4 be the corresponding tree number of the company at the end of the first, second, third and fourth months.

From the assumption we have: $y_1 = 0.9y_0 + 100$,

And thus, $10y_1 - 9y_0 = 1000$.

Using the same analysis, we get $10y_2 - 9y_1 = 1020$, $10y_3 - 9y_2 = 1040$, $10y_4 - 9y_3 = 1060$.

In addition, we have: $y_4 + y_0 = 80$.

So, we have the following equation system:

$$\begin{cases} 10y_1 - 9y_0 = 1000 \\ 10y_2 - 9y_1 = 1020 \\ 10y_3 - 9y_2 = 1040 \\ 10y_4 - 9y_3 = 1060 \\ y_4 + y_0 = 80. \end{cases}$$

This is equivalent to

$$\begin{cases} 10y_1 - 9y_0 = 1000 \\ 10y_2 - 9y_1 = 1020 \\ 10y_3 - 9y_2 = 1040 \\ 10y_0 - 9y_3 = 260 \end{cases}$$

Step 2: So, we have a system of linear equations with the following given matrix:

$$\begin{pmatrix} -9 & 10 & 0 & 0 \\ 0 & -9 & 10 & 0 \\ 0 & 0 & -9 & 10 \\ 10 & 0 & 0 & -9 \end{pmatrix} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1000 \\ 1020 \\ 1040 \\ 260 \end{pmatrix}$$

Thereafter, we obtain

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -9 & 10 & 0 & 0 \\ 0 & -9 & 10 & 0 \\ 0 & 0 & -9 & 10 \\ 10 & 0 & 0 & -9 \end{pmatrix}^{-1} \begin{pmatrix} 1000 \\ 1020 \\ 1040 \\ 260 \end{pmatrix}$$

Step 3: Using the formula for calculating the inverse matrix A^{-1} in the above sentence, we can calculate:

$$y_0 = \frac{729*1000 \ + \ 810*1020 \ + \ 900*1040 \ + \ 1000*260}{19*181} = 800.$$

Thereafter, we get $y_4 = 880$, and conclude that there are 800 trees in the city now.

Solution to Problem 2:

To solve the problem by setting up the system of equations, we proceed through three steps as follows:

Step 1: Let x_1, x_2, x_3, x_4 and x_5 be the product numbers of each type of product A, B, C, D, and E, respectively. To use up the capacity of the plant, we set:

$$\begin{cases} x_1 + 4x_2 + 8x_3 + 12x_4 + 20x_5 = 180, \\ x_1 + 3x_2 + 12x_3 + 15x_4 + 24x_5 = 220, \\ x_1 + 3x_2 + 6x_3 + 10x_4 + 10x_5 = 120, \\ x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 60, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 20. \end{cases}$$

Step 2: So, we have a system of linear equations represented in the following given matrix:

$$\begin{pmatrix} 1 & 4 & 8 & 12 & 20 \\ 1 & 3 & 12 & 15 & 24 \\ 1 & 3 & 6 & 10 & 10 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 180 \\ 220 \\ 120 \\ 60 \\ 20 \end{pmatrix}$$

Thereafter, we get

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 8 & 12 & 20 \\ 1 & 3 & 12 & 15 & 24 \\ 1 & 3 & 6 & 10 & 10 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 180 \\ 220 \\ 120 \\ 60 \\ 20 \end{pmatrix}$$

Step 3: Solve the system of equations above, we get $x_1 = x_2 = x_3 = x_4 = x_5 = 4.$

5.3. Analysis

It can be seen that the practical problems are illustrated in the tests are very interesting and meaningful. Using practical problems in teaching mathematics is a contemporary trend. To help High School teachers, as well as University lecturers, easily access teaching methods combined with practical problems, we propose some solutions as follows:

In the process of preparing lectures, the knowledge is related to reality, it is necessary to include practical problems for High School students as well as University students to see clearly that mathematics is useful in our lives. On that basis, teachers need to build a system of appropriate questions and pose some real-life situations that often occur in life for students to solve themselves.

In the process of teaching a certain subject and forming new knowledge for students, we recommend teachers to conduct activities in the following order: warm-up – knowledge creation - practice activities - Explore and expand activities to help students understand the contents in the lessons easily. Afterward, it is good if teachers can suggest real-life problems for students to solve.

Also, teachers should use some teaching skills to make the classroom atmosphere lively and friendly. They should be approachable for students to express their opinions on practical problems. In the teaching process, it is good if teachers can create interest in learning through games, storytelling, practical activities, and problems associated with real life.

Most importantly, teachers should take their time to guide students in applying mathematical knowledge to solve practical problems that are meaningful to everyday life. Finally, in setting examination questions, teachers should set some practical problems into the rich and diverse content of the questions so that students can apply mathematical knowledge into practice following the innovative spirit of the textbooks.

The approach of Mathematical Modeling Teaching in our paper is very suitable for teaching according to the capacity development orientation according to the general education curriculum in 2018.

6. Conclusion

Matrix theory plays a very important role in teaching Mathematics and solving mathematical problems. Studying the theory of matrix can help academics, practitioners, and students solve many problems in Engineering, Econometrics, Finance, Economics, Decision Sciences, and many other areas. To review the matrix theory with applications, in this paper we first review the theory of matrix. We then discuss how to build up some mathematical, financial, economic, and statistical models by using matrix theory and discuss the applications by using the theory of matrix in Decision Sciences and other related areas like Mathematics, Economics, Finance, Statistics, and Education with real-life examples.

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